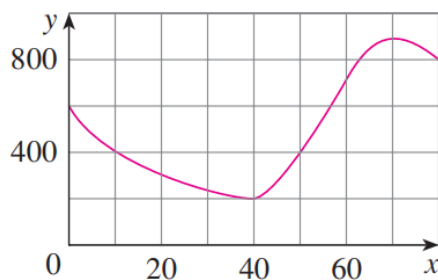


Exercise 18

The graph of a function f is shown.

- Find the average rate of change of f on the interval $[20, 60]$.
- Identify an interval on which the average rate of change of f is 0.
- Which interval gives a larger average rate of change, $[40, 60]$ or $[40, 70]$?
- Compute $\frac{f(40) - f(10)}{40 - 10}$; what does this value represent geometrically?



Solution

Part (a)

The average rate of change on the interval $[20, 60]$ is

$$\frac{f(60) - f(20)}{60 - 20} \approx \frac{700 - 300}{40} = \frac{400}{40} = 100,$$

where $f(60)$ and $f(20)$ are estimated from the given graph.

Part (b)

The average rate of change on the interval $[10, 50]$ is

$$\frac{f(50) - f(10)}{50 - 10} \approx \frac{400 - 400}{40} = \frac{0}{40} = 0.$$

Part (c)

The average rate of change on the interval $[40, 60]$ is

$$\frac{f(60) - f(40)}{60 - 40} \approx \frac{700 - 200}{20} = \frac{500}{20} = 25,$$

whereas the average rate of change on the interval $[40, 70]$ is

$$\frac{f(70) - f(40)}{70 - 40} \approx \frac{900 - 200}{30} = \frac{700}{30} \approx 23.3.$$

Part (d)

The average rate of change on the interval $[10, 40]$ is

$$\frac{f(40) - f(10)}{40 - 10} \approx \frac{200 - 400}{30} = -\frac{200}{30} \approx -6.67.$$

Geometrically, this is the slope of the secant line to the graph over $[10, 40]$.